Math 240 Quiz 2 (1.4, 1.5, 1.7)

NetID: ______________ **Class time:** ______________

Instructions: Calculators, course notes and textbooks are **NOT** allowed on the quiz. All numerical answers **MUST** be exact; e.g., you should write π instead of 3.14..., $\sqrt{2}$ instead of 1.414..., and $\frac{1}{3}$ instead of 0.3333... Explain your reasoning using complete sentences and correct grammar, spelling, and punctuation.
Show ALL of your work!
You have 20 minutes.

Question 1 (2 points). Write down a 3×4 matrix whose columns do *not* span $\mathbb{R}^3$.

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 2 (2 points). Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form, where A is row equivalent to the matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} x_1 = -x_2 - x_4 \\ x_2 = -x_4 \\ x_3 \text{ free} \\ x_4 \text{ free} \end{cases} \Rightarrow \vec{x} = \begin{bmatrix} t-t \\ -t \\ s \\ t \end{bmatrix} = s\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

Question 3 (3 points). True or false? If A is an $m \times n$ matrix with $m > n$, then $A\mathbf{x} = \mathbf{0}$ has at least one nontrivial solution. If true, justify your answer. If false, give a counterexample.

False. Let $A = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \\ \hline & 0 & \end{bmatrix}$.

Then $A\vec{x} = A\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \vec{0}$ forces $\vec{x} = \vec{0}$.

(The statement is true if $m < n$, when there is at least a free variable.)

Question 4 (3 points). Determine if the following vectors are linearly independent. Justify your answer.

$$\underset{\vec{a}}{\begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}}, \quad \underset{\vec{b}}{\begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}}, \quad \underset{\vec{c}}{\begin{bmatrix} 4 \\ 1 \\ -17 \end{bmatrix}}.$$

Suppose $x_1\vec{a} + x_2\vec{b} + x_3\vec{c} = \vec{0}$. We then have

$$\left[\begin{array}{ccc|c} 1 & -2 & 4 & 0 \\ 4 & -3 & 1 & 0 \\ -2 & 7 & -17 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 4 & 0 \\ 0 & 5 & -15 & 0 \\ 0 & 3 & -9 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & 4 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 3 & -9 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 4 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Thus $\begin{cases} x_1 = 2 \\ x_2 = 3 \\ x_3 = 1 \end{cases}$ gives a nontrivial solution.

This implies that the given vectors are not linearly independent.